Bharati Vidyapeeth Institute of Technology

Navi Mumbai

A Workbook for

APPLIED MATHEMATICS

(Mechanical group-22206)
(Computer group-22224)
(Electronics group-22210)

Semester- II

Diploma All Courses

COURSE OUTCOMES (COs)

The theory, practical experiences and relevant soft skills associated with this course are to be taught and implemented, so that the student demonstrates the following industry oriented COs associated with the above mentioned competency:

1. Calculate the equation of tangent, maxima, minima, radius of curvature by differentiation.
2. Solve the different problems of integration using suitable methods.
3. Apply concept of integration to find area and volume.
4. Solve the differential equations of first order and first degree using suitable methods.

FOR MECHANICAL GROUP

5. Utilize basic concepts of probability distribution to solve elementary engg. problems.

FOR COMPUTER GROUP

5. Apply the concept of numerical methods in computer programming languages.

FOR ELECTRONICS GROUP

5. Use Laplace Transform to solve first order and first degree Differential Equations.

List of Tutorials

Academic Year:

Name of Faculty:

Course code:

Subject Code: AMS (22206/22224/22210)

Name of candidate:

Enroll no.

Roll no.

Semester: II

Sr. No.	Title	Courses	Date of performance	Date of submission	Sign of Teacher
1	Solve problems based on finding value of the function.	All			
2	Solve problems to find derivative.	All			
3	Solve problems based on Applications of Derivatives.	All			
4	Solve the problems based on standard formulae of Indefinite Integration.	All			
5	Solve the problems based on standard formulae of Definite Integration.	All			
6	Solve problems based on definite Integration.	All			
7	Solve the problems based on Differential Equations.	All			
8	Solve the problems based on Applications of Differential Equations.	All			
9	Solve the Problems based on Probability.	ME/CH			
10	Solve the Problems based on Probability Distributions.	ME/CH			
11	Solve problems based on Solutions of Algebraic Equations.	CM/IF			
12	Solve problems based on Solutions of Simultaneous Equations.	CM/IF			
13	Solve problems based on algebra of complex numbers	EJ/IS			
14	Find Laplace transform and inverse Laplace transform using related properties.	EJ/IS			

Signature of student **Signature of Faculty**

Chapter 1 (Function)

Q. 1. $If\ f(x) = x^2 + x + 4\ , find\ the\ values\ of\ f(2),\ f(-3),\ f\left(\frac{1}{2}\right) and\ f(0).$

Q. 2. $If\ f(x) = x^3 - 3x^2 + 5\ , find\ the\ values\ of\ f(0) + 3\ f(3).$

Q. 3. $If\ \mathrm{f(x)} = \tan x\,, the\ show\ that\ f(2x) = \frac{2\,f(x)}{1-[f(x)]^2}$

Q. 4. $If\ f(x) = 3x^4 + x^2 + 5 - 3\cos x + 2\ sin^2 x,\ show\ that\ f(x) is\ an\ even\ function.$

Q. 5. $If\ y = f(x) = \frac{x-5}{5x-1},\ show\ that\ f(y) = x.$

Q. 6. $If\ f(x) = \frac{x+5}{3x-4}\ \ and\ t = \frac{5+4x}{3x-1}\ ,show\ that\ f(t) = x.$

Q. 7. $If\ f(x) = \log\left(\frac{x}{x-1}\right), show\ that\ f(a+1) + f(a) = \log\left(\frac{a+1}{a-1}\right).$

Q. 8. $If\ f(x) = x^2 + 4\ and\ f(x+1) - f(x-1) - 12 = 0\ find\ x.$

Q. 9. $If\ f(t) = 50 \sin(100\pi t + 0.4),\ prove\ that\ f\left(\frac{1}{50} + t\right) = f(t).$

Q. 10. $If\ f(x) = \log x\ ,\ g(x) = x^3,\ show\ that\ f[g(x)] = 3f(x).$

Chapter 2 (Derivative)

Q. 1. $Differentiate\ with\ respect\ to\ x$: $y = (x + \frac{1}{x})^2$

Q. 2. $Differentiate\ with\ respect\ to\ x$: $y = e^x \log x$

Q. 3. $Differentiate\ with\ respect\ to\ x$: $y = x^3\ 3^x \sin x$

Q. 4. *Differentiate with respect to x:* $y = \frac{(1+x^2)\tan^{-1}x}{3x-2}$

Q. 5. *Differentiate with respect to x:* $y = \tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$

Q. 6. *Differentiate with respect to x:* $y = \log(\log(\log(x^2 + 1)))$

Q. 7. *Differentiate with respect to x:* $y = (\tan x)^{\cot x}$

Q. 8. $Find\ \frac{dy}{dx}, if\ x = a\ cos^3\theta\ and\ y = x = a\ sin^3\theta$

Q. 9. $Find\ \frac{dy}{dx}, if\ x^3 + 3x^2y + 2xy^2 - y^3 = 7$

Q. 10. $Differentiate\ \sin^{-1}\left(\frac{2x}{1+x^2}\right)\ w.r\ t. \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Q. 11. $Differentiate \log(\tan x)\ \ w.r\ t. sin^{-1}\sqrt{x}.$

Q. 12. $Find\ \frac{d^2y}{dx^2}\ at\ \theta = \frac{\pi}{2}\ if\ x = a(1 - \cos\theta)\ and\ y = a(\theta - \sin\theta).$

Q. 13. $If\ y = a\cos(\log x) + b\sin(\log x)\ \ then\ show\ that\ x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$

Q. 14. $If\ y = e^{m \sin^{-1} x}$ $then\ prove\ that$ $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2 y = 0.$

Chapter 3 (Application of derivative)

Q. 1. *At what point does the curve* $y = x^3 - 24x + 2$ *has slope equal to* 3 ?

Q. 2. *Find the equation of tangent and normal to the curve* $2x^2 - xy + 3y^2 = 18$

at (a, b)

Q. 3. *Show that* $\frac{x}{a}+\frac{y}{b}=2$ *is the tangent to the curve* $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ *at* (a,b)

Q. 4. $Find\ Maximum\ and\ minimum\ values\ of\ the\ function\ y = x^3 - 6x^2 - 15x + 21.$

Q. 5. *A bullet fired into a mud bank penetrates* $(120t - 3600t^2)$ *meter in t second after impact. Calculate maximum depth of penetration.*

Q. 6. $If\ an\ alternating\ current\ is\ given\ by\ I = 50\sin(100at + 0.004).\ Find\ the\ value$

$of\ t\ for\ which\ I\ is\ maximum.\ Also\ find\ the\ maximum\ value\ of\ I.$

Q. 7. $Find\ the\ radius\ of\ curvature\ of\ y = x^3\ at\ (2,8).$

Q. 8. *Find radius of curvature of the curve* $y = \log(\sin x)$ *at* $x = \frac{\pi}{2}$.

Chapter 3 (Indefinite Integration)

Q. 1. $Evaluate: \int \frac{1}{25-9x^2}dx.$

Q. 2. $Evaluate: \int \frac{\cos x}{(1+\sin x)^{3/2}}dx.$

Q. 3. $Evaluate\ :\ \int \frac{e^{x}(1+x)}{\cos^2(xe^{x})}dx.$

Q. 4. $Evaluate\ :\ \int \frac{1}{x\log x\log(\log x)}dx.$

Q. 5. $Evaluate: \int x^2 \log x \, dx.$

Q. 6. $Evaluate: \int \sin^{-1} x \, dx.$

Q. 7. $Evaluate: \int \frac{1}{(x+1)(x+2)(x+3)} dx.$

Q. 8. $Evaluate: \int \frac{dx}{x^3-4x}.$

Q. 9. $Evaluate:\ \int \frac{5x-4}{x^2-8x+12}\,dx.$

Q. 10. $Evaluate: \int \frac{dx}{cos^2x\,(1+\tan x)(1+2\tan x)}$.

Chapter 4 (Definite Integration)

Q. 1. $Evaluate: \int_0^{\pi/2} \sin x \cos x \, dx.$

Q. 2. $Evaluate: \int_0^1 x\,(1-x)^{3/2} \, dx.$

Q. 3. $Evaluate: \int_0^{\pi/2} \frac{\sin x}{(1+\cos x)^2}\,dx.$

Q. 4. $Evaluate: \int_0^{4} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{4-x}}\,dx.$

Q. 5. $Evaluate: \int_{\pi/6}^{\pi/3} \frac{1}{1+\cot x}\, dx.$

Q. 6. $Evaluate: \int_0^{\pi/4} \log(1+\tan x)\, dx.$

Q. 7. $Evaluate: \int_0^{\pi} \frac{x \sin x}{1+cos^2 x} dx.$

Q. 8. $Evaluate: \int_{-\pi/2}^{\pi/2} \frac{sin^3 x}{sin^4 x+cos^4 x} dx.$

Chapter 5 (Application of Definite Integration)

Q. 1. $Find\ the\ area\ bounded\ by\ the\ curve\ y = x^3, x\ axis\ and\ the\ ordinates\ x = 1, x = 3.$

Q. 2. $Find\ the\ area\ bounded\ by\ the\ curve\ y = 4x - x^2, and\ the\ x\ axis.$

Q. 3. *Find the area bounded by the parabola* $y^2 = 16x$ *and its latus rectum.*

Q. 4. *Find the area bounded by the circle* $x^2 + y^2 = 9$ *using integration.*

Q. 5. *Find the area bounded by the curve* $y = x^2$ *and the line* $y = x$.

Q. 6. *Find the area bounded by the curve* $y^2 = 4x$ *and* $x^2 = 4y$.

Q. 7. *Find the area bounded by the curve* $y^2 = 8x$ *and the line* $y = 2x$.

Q. 8. *Find the volume generated by revolving semi* $-$ *circle about its bounding diameter.*

Q. 9. *Find the volume of the solid generated when the triangle bounded by the lines* $y = 0$,

$y = x$ *and* $x = 4$ *is revolved about* $x - axis$.

Q. 10. *Find the volume generated about the* $x - axis$ *the area bounded by* $4y = 3x$, $y = 0$ *and*

$x = 4$.

Chapter 6 (Differential Equation)

Q. 1. *Find the order and degree of the following Differential equations:*

a) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0.$

b) $\frac{[1+\left(\frac{dy}{dx}\right)^2]^{3/2}}{\frac{d^2y}{dx^2}} = 5$

Q. 2. *Form a differential equation by eliminating arbitrary constant from* $y^2 = 4ax.$

Q. 3. $Form\ a\ differential\ equation\ of\ the\ family\ of\ cirlcles\ with\ fixed\ radius\ r\ and$

$whose\ centres\ lies\ on\ x\ axis\ .$

Q. 4. $Solve : (1 + x^3)\ dy - x^2 y\ dx = 0.$

Q. 5. $Solve: (xy^2 + x)\ dx + (yx^2 + y)\ dx = 0.$

Q. 6. $Solve: \frac{dy}{dx} = e^{x-y} + xe^{-y}.$

Q. 7. $Solve : sec^2 x \tan y \, dx + \; sec^2 y \tan x \, dy = 0.$

Q. 8. $Solve : \frac{dy}{dx} = (4x + y + 1)^2.$

Q. 9. $Solve : \frac{dy}{dx} + y \tan x = cos^2 x.$

Q. 10. $Solve : (x+1)\frac{dy}{dx} - y = e^x (x+1)^2.$

Chapter 6 (Application Of differential equations)

Q. 1. $A\ constant\ e.m.f.is\ introduced\ into\ an\ L - R\ circuit.The\ differential\ equation$

$of\ the\ circuit\ is\ E - L\frac{di}{dt} = Ri.Find\ the\ current\ at\ any\ time\ t\ given\ that$

$i = 0\ when\ t = 0.$

Q. 2. $A\ resistance\ of\ 100\ ohms\ and\ an\ inductance\ of\ 0.1\ henries\ are\ connected\ in\ series\ with\ a\ battery\ of\ 20\ volts. Find\ the\ current\ in\ the\ circuit\ as\ a\ function\ of\ time\ t.$

Q. 3. $If\ L.\frac{di}{dt} = 30\sin(10\pi t)\ , find\ i\ in\ terms\ of\ t, fiven\ that\ L = 2\ and\ i = 0\ at\ t = 0.$

Chapter 7 (Probability)

Q. 1. *From a pack of* 52 *cards, one is drawn at random. Find the probability of getting a king.*

Q. 2. *An Urn contains* 10 *black and* 10 *white balls. Find the probability of drawing two balls of the same colour.*

Q. 3. $Find\ the\ probability\ of\ getting\ sum\ of\ 9\ with\ two\ dice.$

Q. 4. $From\ a\ class\ of\ 12\ students, 5\ are\ boys\ and\ rest\ are\ girls. Find\ the\ probability\ a$

$student\ selected\ is\ a\ girl.$

Q. 5. *A room has 3 electric lamps. From a collection of 15 electric bulbs of which only 10 are good, 3 are selected ar random and put in the lamps. Find the probability that the room is lighted by at least one of the bulbs.*

Q. 6. *If A and B are two events such that* $P(A) = \frac{1}{2}, P(B) = \frac{7}{12}$ *find* $P(A' \cap B')$.

Q. 7. $A\ husband\ and\ wife\ appeared\ in\ an\ interview\ for\ two\ vacancies\ in\ an\ office. The$

$probability\ of\ husband's\ selection\ is\ \frac{1}{5}\ and\ that\ of\ wife's selection\ is\ \frac{1}{7}. Find\ the$

$probability\ that\ a)\ both\ of\ them\ are\ selected, and$

$b)\ Only\ one\ of\ them\ is\ selected.$

Q. 8. $Two\ six - faced\ unbiased\ dice\ are\ thrown. Find\ the\ probability\ that\ the\ sum\ of\ numbers\ shown\ is\ 7\ or\ product\ is\ 12.$

Q. 9. *The probability that a student passes HSC examination is* $\frac{2}{3}$ *and the probability that he passes both HSC and IIT entrance examination is* $\frac{14}{45}$. *The probability that he passes at least one examination is* $\frac{4}{5}$. *What is the probability that he passes the IIT entrance examination?*

Q. 10. *A problem is given to the three students Sumit, Amit and Akbar whose chances of solving it are* $\frac{1}{2}, \frac{1}{3}$ *and* $\frac{1}{4}$ *respectively. If they attempt to solve a problem independently, find the probability that the problem is solved by at least one of them.*

Chapter 8 (Probability Distribution)

Q. 1. $A\ fair\ coin\ is\ tossed\ 6\ times, find\ the\ probability\ of\ getting\ exactly\ 4\ heads.$

Q. 2. $If\ the\ chance\ that\ out\ of\ 10\ telephone\ lines\ one\ of\ the\ line\ is\ busy\ at\ any\ instant\ is\ 0.2.$

$What\ is\ the\ chance\ that\ 5\ out\ of\ the\ lines\ are\ busy.$

Q. 3. *Assuming that 2 in 10 industrial accidents are due to fatique. Find the probability that exactly 2 out of 8 accidents will be due to fatique.*

Q. 4. *If 2% of electric bulbs manufectured by a company are defective, find the probability that in a sample of 100 bulbs at the most 2 bulbs are defective.*

Q. 5. $If\ a\ random\ variable\ has\ a\ poiwwon\ distribution\ such\ that\ P(2) = P(3) find\ P(5).$

Q. 6. $fit\ a\ poission\ distribution\ to\ set\ of\ following\ observations:$

x_i	0	1	2	3	4
f_i	122	60	15	2	1

Q. 7. *If the probability of bad reaction from a certain injection is* 0.001, *determine the chance that out of* 2000 *individuals more than two will get a bad reaction.*

$(Given\ e^2 = 7.4)$

Q. 8. $In\ a\ sample\ of\ 1000\ cases, the\ mean\ of\ certain\ test\ is\ 14\ and\ standard\ deviation\ is\ 2.5.$

$Assuming\ the\ distribution\ to\ be\ normal\ find$

$a)\ How\ many\ students\ score\ between\ 12\ and\ 15.$ $b)\ How\ many\ students\ score\ above\ 18.$

$[Given\ A(0.8) = 0.2881, A(0.4) = 0.1554, A(1.6) = 0.4452.]$

Q. 9. $In\ a\ certain\ examination\ 500\ students\ appeared.\ Mean\ score\ is\ 68\ and\ standard$

$deviation\ is\ 8.\ Find\ the\ number\ of\ students\ scoring\ a)\ Less\ than\ 50\ \ b)\ more\ than\ 60.$

$[Given\ A(2.25) = 0.4878, A(1) = 0.3413.]$

Q. 10. $A\ factory\ manufactured\ 2000\ electric\ bulbs\ with\ average\ life\ of\ 2040\ Hrs.\ and$ $S.D.\ of\ 60\ Hrs.\ Assuming\ normal\ distribution\ find\ the\ number\ of\ bulbs\ having\ life$ $more\ than\ 2150\ Hrs.$ $[Given\ A(1.83) = 0.4667]$.

Chapter 7 (Numerical Methods-Solution of Algebraic Equations)

Q. 1. $find\ a\ root\ of\ the\ equation\ x^3 + 2x - 1 = 0\ in\ the\ interval\ (0, 1)\ using\ the\ Bisection\ method\ in\ three\ iterations.$

Q. 2. *Using Bisection method, find the approximate value of* $\sqrt{10}$ *by performing two iterations.*

Q. 3. $find\ a\ positive\ root\ of\ the\ equation\ x = e^{-x}\ using\ the\ Bisection\ method$

$(perform\ two\ iterations.)$

Q. 4. *Using False position method, find the root of the equation* $x^2 + x - 3 = 0$ *in the interval* $(1, 2)$ *by performing two iterations.*

Q. 5. $Using\ False\ position\ method, find\ the\ approximate\ value\ of\ \sqrt{6}\ by\ performing\ two$

$iterations.$

Q. 6. *By using Regula Falsi method, find the root correct to two decimal places of the equation* $x \log_{10} x = 1.2$ *that lies between 2 and 3.*

Q. 7. *Using Newton Raphson method, find the root of the equation* $x^3 - x - 1 = 0$ *upto two iterations by taking initial root* 1.

Q. 8. *Using Newton Raphson method, find the root of the equation* $x^3 - 5x + 3 = 0$ *in* $(0, 1)$.

by performing two iterations.

Q. 9. $Using\ Newton\ Raphson\ method, evaluate\ \sqrt[3]{100}\ by\ performing\ two\ iterations.$

Q. 10. *Using Newton Raphson method, find the value of* $\frac{1}{\sqrt{12}}$ *by performing two iterations.*

<u>Chapter 8 (Numerical Methods-Solutions of Simultaneous Equations)</u>

Q. 1. $Solve\ the\ following\ equations\ by\ Gauss\ Elimination\ method:$

$$2x + 3y + z = 13, \qquad x - y - 2z + 1 = 0, \qquad 3x + y + 4z = 15$$

Q. 2. $Solve\ the\ following\ equations\ by\ Gauss\ Elimination\ method:$

$x + 2y + 3z = 14, \qquad 3x + y + 2z = 11, \qquad 2x + 3y + z = 11$

Q. 3. *Solve the following equations by Gauss Elimination method :*

$$x + y + z = 6, \qquad 2x + y + 3z = 13, \qquad 2x + 3y + 4z = 20$$

Q. 4. $Solve\ the\ following\ equations\ by\ Jacobi's\ method:$

$$20x + y - 2z = 17, \qquad 3x + 20y - z + 18 = 0, \qquad 2x - 3y + 20z = 25$$

Q. 5. *Solve the following equations by Jacobi's method* :

$$5x + 2y + z = 12, \qquad x + 4y + 2z = 15, \qquad x + 2y + 5z = 20$$

Q. 6. $Solve\ the\ following\ equations\ by\ Jacobi's\ method$:

$$2x + y + z = 4, \qquad x + 2y + z = 4, \qquad x + y + 2z = 4$$

Q. 7. *Solve the following equations by Gauss Seidal method :*

$$x + 7y - 3z + 22 = 0, \qquad 5x - 2y + 3z = 18, \qquad 2x - y + 6z = 22$$

Q. 8. *Solve the following equations by Gauss Seidal method :*

$x + 3y + 10z = 24, \qquad 28x + 4y - z = 32, \qquad 2x + 17y + 4z = 35$

Q. 9. *Solve the following equations by Gauss Seidal method :*

$$3x + 3y - z = 11, \qquad 2x - y + 2z = 9, \qquad 4x + 3y + 2z = 25$$

Q. 10. *Solve the following equations by Gauss Seidal method* :

$$6x + y + z = 105, \qquad 4x + 8y + 3z = 155, \qquad 5x + 4y - 10z = 65$$

Chapter 7 (Complex Number)

Q. 1. $Express\ in\ polar\ form:\ \frac{(1+i)^2}{1-i}$

Q. 2. $Find\ Modulus\ and\ amplitude\ of -\frac{1}{2}+\frac{\sqrt{3}}{2}i\ \ and\ express\ in\ polar\ form.$

Q. 3. *Prove that* $(1+i)^8 + (1-i)^8 = 32.$

Q. 4. *If* $z_1 = -3+4i,\ z_2 = 5-3i,\ find\ \frac{z_1}{z_2}\ and\ \frac{1}{z_1}+\frac{1}{z_2}$

Q. 5. $If\ (3+i)x + (1-i)y = 1 + 7i,\ find\ x\ and\ y.$

Q. 6. $If\ z_1 = 4 + 5i,\ z_2 = 3 + 7i,\ find\ |3z_1 - 2z_2|\ \ and\ \ |z_1| + |z_2|$

Q. 7. $Find\ Modulus\ and\ Amplitude\ of\ 1 - \cos\theta + i\sin\theta$

Q. 8. $By\ using\ De - Moivre's\ theorem, simplify\ \frac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 4\theta - i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3 (\cos 5\theta + i\sin 5\theta)^4}$

Q. 9. *If n is positive integer, prove that* : $(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1}\cos(\frac{n\pi}{6})$

Chapter 8 (Laplace transform)

Q. 1. $Find\ L[\sin 3 + \cos 2t]$

Q. 2. $Find\ L[e^{4t} \cos 5t]$

Q. 3. $Find\ L[cos^3 2t]$

Q. 4. $Find\ L[t\ \sin 4t]$

Q. 5. $Find\ L[\sin 3t \cos 2t]$

Q. 6. $Find\ L[t\, e^{2t} \cos 3t]$

Q. 7. $Find\ L^{-1}[\frac{s}{s^2+2}]$

Q. 8. $Find\ L^{-1}[\frac{s+2}{s^2+4s+13}]$

Q. 9. $Find\ L^{-1}[\frac{s^2+3s+2}{s^3}]$

Q. 10. $Find\ L^{-1}[\frac{2s+3}{(s+1)(s+2)}]$

Q. 11. $Solve\ \frac{dy}{dx} + 3y = 2 + e^t\ \ if\ y = 1\ at\ t = 0\ using\ Laplace\ transform.$